On the Predictability of Large Transfer TCP Throughput

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Abstract— With the advent of overlay and peer-to-peer networks, Grid computing, and CDNs, network performance prediction becomes an essential task. Predicting the throughput of large TCP transfers, in particular, has attracted much attention. In this work, we focus on the design, empirical evaluation, and analysis of TCP throughput predictors for a broad class of applications. We first classify TCP throughput prediction techniques into two categories: Formula-Based (FB) and History-Based (HB). Within each class, we develop representative prediction algorithms, which we then evaluate empirically over the RON testbed. FB prediction relies on mathematical models that express the TCP throughput as a function of the characteristics of the network path (e.g., RTT, loss rate, available bandwidth). FB prediction does not rely on previous TCP transfers in the given path, and it can be performed with non-intrusive network measurements. We show, however, that the FB method is accurate only if the TCP transfer is window-limited to the point that it does not saturate the underlying path, and explain the main causes of the prediction errors. HB techniques predict the throughput of TCP flows from a time series of previous TCP throughput measurements on the same path, when such a history is available. We show that even simple HB predictors, such as Moving Average and Holt-Winters, using a history of limited and sporadic samples, can be quite accurate. On the negative side, HB predictors are highly path-dependent. Using simple queueing models, we explain the cause of such path dependencies based on two key factors: the load on the path, and the degree of statistical multiplexing.

Keywords: Network measurements, TCP throughput, time series forecasting, performance evaluation

I. Introduction

With the advent of overlay and peer-to-peer networks [4], [10], Grid computing [12], and CDNs [19], performance prediction of network paths becomes an essential task. To name just a few of their applications, such predictions are used in route selection schemes for overlay and multihomed networks [2], [3], [5], [21], dynamic server selection [25], [26], [38], and peer-to-peer parallel downloads [9].

Arguably, the most important performance metric of a path is the average throughput of TCP transfers. The reason is that most data-transfer applications, and about 90% of the Internet traffic, use the TCP protocol. When it comes to performance prediction, the focus is typically on bulk TCP transfers, lasting more than a few seconds. Short TCP flows are often limited by slow-start, and their performance is much more sensitive to the randomness in the background traffic [13]. In this work, we focus on predicting the throughput of a bulk TCP transfer in a particular network path, *prior* to actually starting the flow. For many applications, such as server selection and overlay route selection, a throughput prediction is needed before the flow starts. The reason is that rerouting an established TCP connection to a different network path or server can cause problems such as migration delays, packet reordering, re-initialization of

the congestion window. Note that TCP throughput prediction is different than TCP throughput *estimation*. The latter is performed while the flow *is in progress* with the objective to estimate the TCP throughput or the TCP-Friendly rate of the flow. An example of a TCP throughput estimation scheme is TCP-Friendly Rate Control (TFRC) [11].

Unlike the prediction of RTT and loss rate, which can be based on direct and low-overhead measurements, predicting TCP throughput is significantly harder. First, TCP throughput depends on a large number of factors, including the transfer size, maximum sender/receiver windows, various path characteristics (RTT, loss rate, available bandwidth, the nature of cross traffic, reordering, router/switch buffering, and others) and the exact implementation of TCP at the end-hosts. Second, direct measurement of TCP throughput using large "probing" transfers can be highly intrusive because the latter can saturate the underlying paths for significant time periods. What is really desired is *a low-overhead TCP throughput prediction technique that either avoids probing transfers altogether, or requires only a limited amount of probing traffic.*

In this paper, we focus on the design, empirical evaluation, and analysis of TCP throughput predictors for a broad class of applications. The common requirement of such applications is that they rely on an accurate throughput prediction prior to the start of the TCP transfer. We first classify TCP throughput prediction techniques into two categories: *Formula-Based* (FB) and *History-Based* (HB). Within each class we develop representative prediction algorithms, which we then evaluate empirically over the RON testbed [1]. Note that our objective is not to compare FB and HB predictors. In fact, the two schemes are complementary, as they require different types of measurements and previous information about the underlying path. Instead, our objective is to examine the key issues in each prediction scheme, evaluate their accuracy under different conditions, explain the major causes of prediction errors, and provide insight regarding the factors that affect the predictability of large transfer TCP throughput in a given path.

More specifically, FB prediction relies on mathematical models that express the TCP throughput as a function of the characteristics of the underlying network path (e.g., RTT, loss rate). For instance, the throughput-optimizing routing component of RON follows the FB approach [4], predicting TCP throughput based on the simple "square-root" formula of [20]. That formula expresses the average throughput of a congestion-limited bulk transfer as a function of the RTT and the loss rate that the connection experiences on a given path. Several similar models have been proposed in the literature [6], [8], [14], [22], [30], differing in terms of complexity and accuracy, modeling assumptions, and TCP flavor. In this paper,

we prefer to use the main result of [22], referred to as the *PFTK formula*, because it is both simple and quite accurate.

The main advantage of FB prediction is that it does not require any history of previous TCP transfers. In addition, FB prediction can be performed with relatively lightweight, non-intrusive network measurements of parameters such as RTT and loss rate. Unfortunately, however, our measurements show that FB schemes can lead to large prediction errors. The main reason is that throughput models require knowledge of the path characteristics *during* the TCP flow, whereas FB predictions measure the corresponding a priori characteristics *before* the flow starts. If the flow itself causes significant changes in those characteristics, the resulting prediction errors can be unacceptably large. Another reason is that the delays or losses that a TCP flow experiences are not necessarily the same as those observed by a periodic probing stream, such as *ping* [15]. On the positive side, we do observe that the prediction errors are much lower, and probably acceptable for most applications, if the TCP transfer is limited by the receiver's advertised window to the point that the transfer does not saturate its path.

On the other hand, HB approaches use standard time series forecasting techniques to predict TCP throughput based on a history of throughput measurements from previous TCP transfers on the same path. Obviously, HB prediction is applicable only when large TCP transfers are performed repeatedly on the same path. This is the case with several applications of TCP throughput prediction, including overlay network routing, parallel downloading and Grid computing.

Our measurements over the RON testbed show that even simple linear HB predictors, such as Moving Average and non-seasonal Holt-Winters, are quite accurate. Furthermore, in agreement with previous work on HB prediction [34], [39], we found no major differences among a few candidate HB predictors. We do find, however, that two simple heuristics can noticeably improve the accuracy of HB predictors. The first is to detect and ignore outliers, and the second is to detect level shifts and restart the HB predictors. We next show, perhaps surprisingly, that even with a short history of a few previous transfers performed sporadically in intervals up to 30-40 minutes, prediction errors are still fairly low. On the negative side, our measurements show that HB predictors are highly path-dependent, which begs for answers to the following two questions. What makes TCP throughput much more predictable on some paths than on others, and which are the fundamental factors that affect the throughput predictability on a path? Using simple queueing models, we focus on two factors that we believe are the most important: the load on the path, and the degree of statistical multiplexing. Specifically, we show that the prediction error increases with the load on the bottleneck link, and decreases with the number of competing flows under constant load. Consequently, paths that are heavily loaded with just a few big flows are expected to be most difficult to predict.

The structure of the paper is as follows. We summarize the related work in Section 2. In Section 3, we develop a representative FB predictor and highlight some important issues in that type of prediction. Section 4 presents measurement results for the accuracy of FB prediction. Section 5 introduces several existing HB predictors, and describes two simple techniques that can improve such predictors significantly. Section 6 presents measurement results for the accuracy of HB prediction. Section 7 focuses on two major factors that affect the throughput predictability: the load of the network path, and the degree of statistical multiplexing. We conclude in Section 8.

II. RELATED WORK

One motivation for some of the previous work on TCP throughput modeling has been to predict the throughput of a transfer as a function of the underlying network characteristics [11], [20], [22]. However, the accuracy of FB prediction depends on the accuracy with which these characteristics can be estimated or measured. Recently, Goyal et al. have shown that the end-to-end packet loss rate p on a path can be quite different from the "congestion event probability" p' required by the well-known PFTK model by Padhye et al. [22], and they have proposed a way to estimate p' from p [15]. Note that that work does not address the problem of estimating the required path characteristics during a flow from those observed prior to the flow.

HB TCP throughput prediction has been previously studied, mostly in the context of Grid computing [32], [34], [35], [36]. One operational system is the Network Weather Service (NWS) project [37]. In NWS, throughput prediction is based on small (64KB) TCP transfer probes with a limited socket buffer size (32KB). Vazhkudai et al. use bulk TCP transfers (1MB-1GB) and a large socket buffer (1MB), performed sporadically (1 minute-1 hour) [34]. They show that various linear predictors (including ARIMA models) perform similarly, and that the average prediction error on two paths ranges from 10% to 25%. Zhang et al. examine TCP throughput predictability based on a large set of paths and transfers [39]. Their TCP throughput measurements use 1MB transfers performed every minute, with 200KB socket buffers. Their main results are that 1) with several simple linear predictors, about 95% of the prediction errors are below 40%, and 2) predictions using a very long history (e.g., Moving Average with 128 samples) perform rather poorly. A study by Qiao et al. has shown that the predictability of network traffic is highly path dependent [24]. Also, mathematical models (such as MMPP) that have been previously used to analyze the predictability of aggregate network traffic [29] are not directly applicable to the predictability of TCP throughput.

Even though the previous work on HB prediction is substantial, it left three important open issues that we attempt to address with this work. First, it did not distinguish between congestion-limited and window-limited flows; typically, the latter have a small socket buffer at the receiver or sender compared to the path's bandwidth-delay product, and so they do not impose a heavy load on the network. Second, the previous work did not examine the effect of the TCP transfer frequency on the HB prediction accuracy; this frequency is a crucial parameter for HB prediction. Third, previous work has not investigated the underlying path characteristics that determine the predictability of TCP throughput. Instead, the

underlying network path has been viewed as a "black box", and so it was not possible to relate its characteristics (such as load and degree of statistical multiplexing) to the resulting TCP throughput predictability.

III. FORMULA-BASED PREDICTION

The central component of an FB predictor is a mathematical formula that expresses the average TCP throughput as a function of the underlying path characteristics. Probably the most well-known such model is the "square-root" formula of [20]:

$$E[R] = \frac{M}{T\sqrt{\frac{2bp}{3}}} \qquad (1)$$

where $E[R]$ is the *expected TCP throughput* (as opposed to R which denotes the *actual or measured throughput* and $\hat{R}$ which denotes the *predicted throughput*). In the previous formula, M is the flow's Maximum Segment Size, b is the number of TCP segments per new ACK, while T and p are the RTT and loss rate, respectively, as experienced by the TCP flow[1]. This model is fairly accurate for bulk TCP transfers in which packet losses are recovered with Fast-Retransmit. Analytical results such as (1) have been very useful in understanding the relation between TCP throughput and certain key path characteristics, such as loss rate and RTT.

Another motivation for the research that led to these TCP models was the ability to *predict* the throughput of a TCP flow given estimates of the relevant path characteristics [8], [15], [22]. In this section, we first present a more complete TCP throughput formula, as well as the corresponding FB predictor. Although several similar models exist in the literature (see [30] and the references therein), we emphasize that our remarks regarding the accuracy and limitations of FB prediction are not specific to the particular formula we use.

A. A formula-based TCP throughput predictor

The TCP throughput formula that we use is the PFTK result of [22], which improves on the square-root formula especially in the presence of retransmission timeouts and/or a limited maximum window:

$$E[R] = \min\left(\frac{M}{T\sqrt{\frac{2bp}{3}} + T_o\min(1,\sqrt{\frac{3bp}{8}})p(1+32p^2)}, \frac{W}{T}\right) \qquad (2)$$

where T_o is the TCP retransmission timeout period, and W is the maximum window size (limited by the socket buffer size at the sender or receiver). We emphasize that p and T, in the previous equation, are the average loss rate and RTT that the *target flow* (i.e., the TCP flow whose throughput we try to predict) experiences. Notice that the loss rate p may be zero, in which case the flow is *lossless* and $E[R]$ is given by the term W/T.

Suppose now that we want to apply (2) to TCP throughput prediction. The main problem is that we do not know, when

[1]The main mathematical symbols we use are summarized in the Appendix at the end of the paper.

predicting, the loss rate and RTT that the flow will experience during its lifetime. The obvious approach, which has been previously followed in practice (e.g., in overlay routing [4]), is to measure the loss rate and RTT *before* the transfer with a utility such as *ping*, and then apply those estimates of p and T in (2). Suppose that $\hat{p}$ and $\hat{T}$ are the loss rate and RTT estimates based on measurements prior to the flow. Then, if $\hat{p} \approx p$ and $\hat{T} \approx T$, the prediction accuracy will be only limited by the accuracy of these approximations and the accuracy of the mathematical model that was used to derive (2). We can expect that $\hat{p} \approx p$ and $\hat{T} \approx T$ when the TCP flow imposes a minor load on the path's bottleneck, and so it does not affect significantly the RTT and loss rate of the path.

A limitation of the previous approach is that it does not apply to *lossless paths*, i.e., when $\hat{p}=0$. In that case, $W/\hat{T}$ can be totally unrelated to the realized throughput, especially if W is much larger than the bandwidth-delay product of the underlying path. One approach to deal with lossless paths is to predict the TCP throughput based on the *available bandwidth* $\hat{A}$ of the path prior to the TCP flow, when $\hat{A} < W/\hat{T}$. The available bandwidth is the non-utilized part of the bottleneck link's capacity, and it can be measured non-intrusively with end-to-end probing techniques [16], [18], [27], [31]. Although the available bandwidth and TCP throughput are not expected to be exactly equal, $\hat{A}$ can be used as a first-order approximation of R when the flow is not limited by its maximum window size W [16]. On the other hand, if $W/\hat{T} < \hat{A}$, the flow cannot obtain all the available bandwidth due to its limited maximum window, so $W/\hat{T}$ is a more reasonable predictor; we refer to such flows as *window-limited*.

To summarize, the FB predictor that we consider in the rest of this paper is given by the following equation:

$$\hat{R} = \begin{cases} \min\left(\frac{M}{\hat{T}\sqrt{\frac{2b\hat{p}}{3}} + \hat{T}_o\min(1,\sqrt{\frac{3b\hat{p}}{8}})\hat{p}(1+32\hat{p}^2)}, \frac{W}{\hat{T}}\right) & \text{if } \hat{p} > 0 \\ \min\left(\frac{W}{\hat{T}}, \hat{A}\right) & \text{if } \hat{p} = 0 \end{cases}$$
$$(3)$$

where $\hat{R}$ is the predicted throughput, while $\hat{T}$, $\hat{p}$, and $\hat{A}$, are the measured RTT, loss rate, and available bandwidth prior to the TCP flow. We estimate the retransmission timeout period as

$$\hat{T}_o = \max(1\text{sec}, 2\text{SRTT}) \qquad (4)$$

where SRTT is set to the measured RTT $\hat{T}$ prior to the target flow. Note the differences between (2) and (3): the latter relies on the estimates $\hat{T}, \hat{p}, \hat{T}_o$, rather than on the actual values T, p, T_o, and it also has a component that depends on the available bandwidth estimate $\hat{A}$.

In the following, we explain three important limitations of the above predictor using basic insight and simple *ns2* simulation scenarios.

B. Effect of the extra load due to the target flow

Basic queueing theory tells us that an increase in the utilization of a queue (with non-periodic arrivals) increases the average queueing delay. Similarly, in a queue with a limited buffer, an increase in the utilization can cause a higher loss probability. The increase in the queueing delays and/or

the loss probability tends to be more significant when the utilization becomes significantly higher, or when the utilization was already high even before the additional load.

These basic facts can cause major errors in FB prediction. The reason is that the RTT $\hat{T}$ measured prior to the target flow will not reflect the queueing delay during that transfer. So, $\hat{T}$ can be lower than the RTT T that the target flow experiences. Similarly for the loss rate, it can be that $\hat{p} < p$. The net result of either effect is that the FB predictor can overestimate the TCP throughput, especially when the target flow increases the utilization of the bottleneck significantly or when the latter is already very high. Note that the experimental validation of the PFTK result, reported in [22], was based on the "posthumous" estimation of p and T, i.e., from *tcpdump* packet traces collected at the sender/receiver while the target flow was in progress. Of course the same approach is not possible in the prediction context.

Simulation	C (Mbps)	B (pkts)	$\hat{T}$ (ms)	$\tilde{T}$ (ms)	$\hat{R}$	R
1	20	200	16.7	23.4	13.3	9.2
2	50	200	20.1	20.1	10.6	10.6

(A)

Simulation	C (Mbps)	B (pkts)	$\hat{p}$ (%)	$\tilde{p}$ (%)	$\hat{R}$	R
1	10	50	0.14	0.46	9.2	4.6
2	100	300	0.2	0.22	6.2	6.1

(B)

TABLE I
(A) INCREASED RTT (B) INCREASED LOSS RATE.

We use simple $ns2$ simulations to demonstrate how such prediction errors could take place. The simulations use a simple dumbbell topology with the bottleneck link (capacity C, buffer space B) in the center, and the TCP flows (both the cross traffic and the target flow) traversing that link. Table I-(A) gives an example of the discrepancy between the path's RTT $\hat{T}$ prior to the target flow and the RTT $\tilde{T}$ during the target flow. The target flow as well as a single cross traffic TCP flow are window-limited (W=30KB) so that the bottleneck does not experience any packet losses (p=0). Note that the capacity C in Simulation-1 is lower than in Simulation-2, causing a higher utilization in the former. Specifically, the utilization in Simulation-1 increases from 72% to 98.5% after the target flow starts; the corresponding utilization increase in Simulation-2 is from 24% to 48%. As a result, $\tilde{T}$ is significantly different than $\hat{T}$ in Simulation-1, causing a substantial prediction error. The prediction is very accurate, on the other hand, in Simulation-2 because the utilization remained relatively low even after the target flow started.

Similarly, Table I-(B) exemplifies the discrepancy between the loss rate during the target flow $\tilde{p}$ and the loss rate prior to the target flow $\hat{p}$. The cross traffic is a single TCP flow in Simulation-1, and 15 TCP flows in Simulation-2. None of the flows, including the target flow, are window-limited. Here, we see a large loss rate increase in Simulation-1 after the target flow starts. The loss rate increase is much smaller in Simulation-2 because the buffer size B is significantly larger in that case. As a result, the prediction error is substantial in Simulation-1 but minor in Simulation-2.

C. Errors due to the TCP sampling behavior

Even when the target flow does not affect significantly the path's RTT and loss rate, it is still hard to estimate the RTT and loss rate that the TCP target flow experiences. TCP reduces its packet transmission rate when it experiences losses, which means that it tends to "sample" the RTT and packet loss processes less frequently when the path is congested. This is a very different sampling behavior than that of a utility such as ping, which typically sends periodic probing packets. Also, TCP tends to send bursts of data packets when self-clocking fails (e.g., due to ACK compression), which also leads to a different sampling behavior than periodic probing.

To make things more complex, a mathematical model for the TCP throughput may be based on certain assumptions that affect the interpretation of parameters such as T or p. For instance, the PFTK model assumes that T is constant, and that when a packet is dropped all the remaining packets in that "flight" are also dropped (referred to as a "congestion event"). As a result, the parameter p in (2) should not be the unconditional loss probability among all packets of the target flow, but the congestion event probability. The discrepancy between these two parameters was one of the main focus points in [15].

Table II shows three different "loss rates", all obtained from the same simulation as in Table I-(B). In this Table, $\tilde{p}$ is a ping-based estimate of the loss rate measured with periodic probing packets (40 bytes every 100ms) during the target flow, p is the (unconditional) loss rate that the target flow experienced, and p' is the congestion event probability estimated from a detailed analysis of the $ns2$ packet trace. Notice the striking difference, more than an order of magnitude, between $\tilde{p}$ and the other two metrics. Ping estimates a larger loss rate, due to its non-adaptive sampling behavior that we mentioned earlier. The difference between p and p' is also noticeable, although not major. Unfortunately, it is not known how to measure p' or p *prior* to the start of the target flow. For this reason, existing FB prediction schemes use ping-based loss rate estimates, which are also much simpler to obtain.

Simulation	$\tilde{p}$	p	p'
1	0.04	0.0046	0.0028
2	0.03	0.0022	0.0015

TABLE II
DIFFERENT LOSS RATE ESTIMATES DURING A TCP FLOW.

D. Errors due to the difference between available bandwidth and TCP throughput

As previously mentioned, when $\hat{p}$=0 and $\hat{A} < W/\hat{T}$, we predict the throughput of the target flow based on the path's available bandwidth $\hat{A}$ prior to that flow. These two metrics, however, can be significantly different in certain cases [18].

First, whether a TCP flow can saturate the available bandwidth of a path depends on the buffer space B at the bottleneck. If B is not sufficiently large, packet losses can cause significant underutilization and the resulting TCP throughput

can be lower than $\hat{A}$. Second, if the competing cross traffic at the bottleneck is made of elastic flows (e.g., persistent TCP flows), the target flow can capture more than $\hat{A}$, by receiving some of the bandwidth previously occupied by cross traffic flows. The actual difference between available bandwidth and TCP throughput in that case depends on the number and the RTTs of the competing TCP flows.

Consequently, the available bandwidth $\hat{A}$ prior to the target flow can cause either overestimation or underestimation of the flow's throughput, depending on the amount of buffering and the "elasticity" of the cross traffic in the path. Given that it is hard to infer network buffering and cross traffic elasticity in practice, it is unclear whether we can design a better FB predictor than $\hat{A}$ in the case of lossless paths.

IV. FB PREDICTION ACCURACY

The previous section argued that FB prediction can be inaccurate under certain network conditions. In this section, we show measurement results from several Internet paths that quantify the inaccuracy of FB prediction, and further analyze these prediction errors. First, we describe the measurement methodology and the dataset we use throughout this paper.

A. Overview of measurement methodology

Our measurements were collected on 35 Internet paths that interconnect nodes of the RON testbed [1][2]. The RON nodes that we used are located mostly in US universities, but there are also two nodes in Europe and one in Korea. Out of the 35 paths, five are transatlantic paths, one between Korea and New York-NY, and the rest within the US. Most paths can deliver at least 10Mbps, but seven of the paths had a DSL or T-1 bottleneck.

We collected seven measurement "traces" on each path, with a total of 245 traces across all paths. Each trace consists of 150 back-to-back measurement "epochs". An epoch starts with an available bandwidth measurement using pathload, followed by a 60-sec measurement of $\hat{p}$ and $\hat{T}$ using a homespun ping utility that generates a 41-byte probing packet every 100ms, followed by a 50-sec TCP transfer (target flow) generated by IPerf [17] (see Figure 1). RTT and loss rate estimates are also measured during the TCP transfer. A 50-second transfer on these paths is long enough to ensure that the flow spends a negligible fraction of its lifetime in the initial slow-start. A total of 36750 TCP transfers (in the same number of epochs) were performed. The duration of each epoch (and also the time interval between successive TCP transfers) was about 2-3 minutes, while the duration of each trace was about 6 hours. The measurements were collected during a week in May 2004.

IPerf allows us to directly control the maximum TCP window size W by limiting the receiver socket buffer size. Unless otherwise noted, we used $W=1$MB, which is large enough to saturate all the paths we experimented with and cause congestion. To examine the effect of W, we also performed the same measurements with $W=20$KB, which, as will be shown

[2]We prefer RON instead of PlanetLab, because the latter is often too heavily loaded for accurate network measurement.
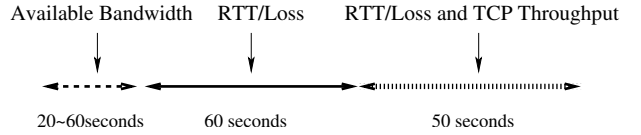

Fig. 1. A measurement epoch. 150 such epochs were recorded during each trace, with 7 traces collected per path.

later, limits the transfer to only a fraction of the available bandwidth on most paths.

Each epoch provides the following measurements: the pre-transfer estimates $\hat{p}$, $\hat{T}$, $\hat{A}$, the actual TCP throughput R, and the estimates of the loss rate $\tilde{p}$ and RTT $\tilde{T}$ *during* the transfer. The first three estimates are used in (3) to predict the TCP throughput $\hat{R}$, which is then compared with the actual throughput R. We collected $\tilde{p}$ and $\tilde{T}$ in order to evaluate how the corresponding metrics change due to the target flow, and also to quantify the prediction error if it was possible to estimate $\tilde{p}$ and $\tilde{T}$ before the target flow.

We define the *relative prediction error E* of an individual measurement epoch as

$$E = \frac{\hat{R} - R}{\min(\hat{R}, R)} \tag{5}$$

Notice that the denominator $\min(\hat{R}, R)$ gives E the property that overestimation or underestimation by the same factor $w > 1$, i.e., $\hat{R}=wR$ for the former and $\hat{R}=R/w$ for the latter, yields the same relative error $w - 1$ (in absolute value).

To report a single figure for n measurements in a time series (specifically, for all the 150 epochs of a trace), we use the *Root Mean Square Relative Error (RMSRE)* statistic, defined as

$$\text{RMSRE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} E_i^2} \tag{6}$$

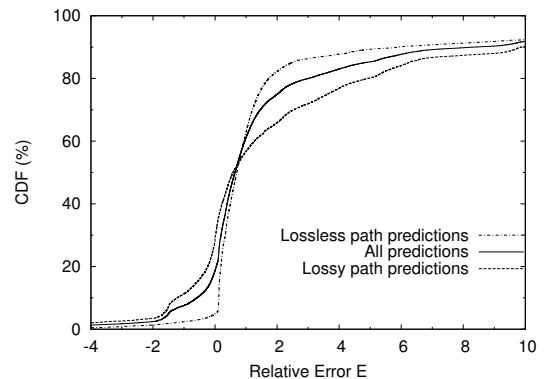where E_i is the relative error of measurement i.

B. Results

Fig. 2. CDF of E for all predictions, for predictions in lossy paths, and for predictions in lossless paths.

Prediction error in lossy and lossless paths: Figure 2 shows the CDF of E for all measurements, across all traces

and paths. It also shows separately the CDFs of E for the subset of lossy path predictions (based on the PFTK model) and for the subset of lossless path predictions (based on the available bandwidth estimate $\hat{A}$)[3]. Let us first focus on the "all predictions" curve. Notice that *for roughly 40% of all measurements, the prediction is an overestimation by more than a factor of two ($E \geq 1$)*. In fact, the overestimation errors are larger than an order of magnitude ($E \geq 9$) for almost 10% of the measurements. The underestimation errors are much less dramatic and common, with only 10% of the measurements suffering from an underestimation by more than a factor of two ($E < -1$).

In the case of lossless paths, underestimation errors occur very rarely, while the overestimation errors are considerably lower and less common than in lossy paths. The reason is that in lossless paths, our FB predictor does not rely on the erroneous RTT and loss rate estimates prior to the target flow. The remaining errors can be attributed to the differences between TCP throughput and available bandwidth, discussed in § III-D. The fact that, in lossless paths, overestimation is the only major type of prediction error implies that either pathload overestimates the path's available bandwidth, or that TCP cannot saturate the available bandwidth in its path due to random losses or insufficient buffering at the bottleneck link.
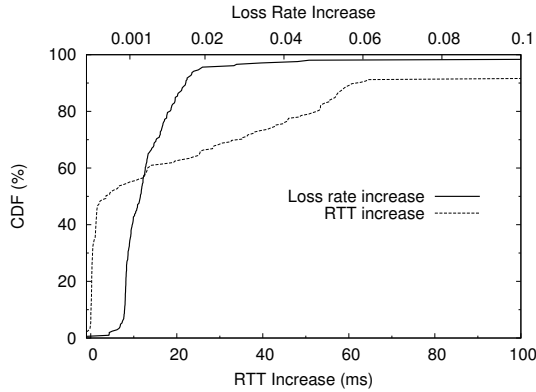

Fig. 3.   CDF of RTT and loss rate increase due to target flow.

RTT and loss rate increases due to target flow: Returning to the case of lossy paths, the fact that overestimation is much more dramatic than underestimation illustrates the dominance of the issue discussed in § III-B, namely $\hat{T} < T$ and $\hat{p} < p$. Figure 3 shows the distributions of the increases in RTT and in loss rate after the start of the target flow. The increases were measured as $\tilde{T} - \hat{T}$ and $\tilde{p} - \hat{p}$ respectively (recall that $\tilde{T}$ and $\tilde{p}$ are estimates of T and p during the target flow). Notice that in about 50% of the measurements, the RTT did not increase significantly. In 40% of the measurements, however, the target flow caused an RTT increase between 5ms and 60ms. In 10% of the measurements the RTT increase was higher than 100ms, probably due to congested low-capacity links (DSL or T-1). The loss rate, on the other hand, increased by 0.1% to 2% in almost all measurements. Even though this loss rate increase may appear small in magnitude, recall that TCP throughput is

inversely proportional to the square-root of the loss rate (see (1)). For example, an increase of the loss rate from 0.1% to 1% can cause a throughput overestimation by a factor of about 3.2.
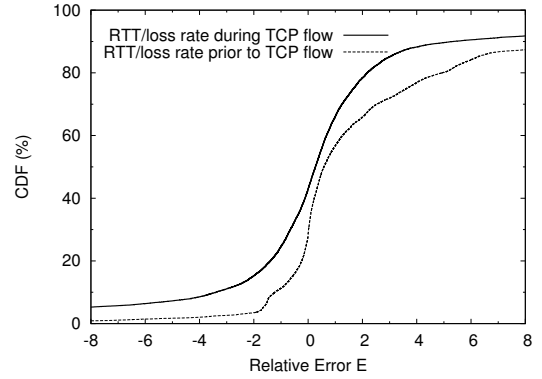

Fig. 4.   Prediction errors using $\tilde{T}$ and $\tilde{p}$ (RTT and loss-rate during the target flow) and using $\hat{T}$ and $\hat{p}$ (RTT and loss-rate prior to the target flow).

Errors due to periodic RTT and loss rate sampling: An interesting hypothetical question is the following: *how accurate would FB prediction be, if we had the estimates of the path's RTT $\tilde{T}$ and of the loss rate $\tilde{p}$ during the target flow?* In theory, it may be possible to estimate $\tilde{T}$ and $\tilde{p}$, given $\hat{T}$ and $\hat{p}$, based on a model that captures the impact of the target flow on the queue of the bottleneck link. Figure 4 shows the CDF of the FB prediction error when we feed in (3) the ping-based RTT $\tilde{T}$ and loss-rate $\tilde{p}$ during the target flow. The CDF refers only to lossy paths. Note that using $\tilde{T}$ and $\tilde{p}$ makes the relative error significantly lower than using $\hat{T}$ and $\hat{p}$ ($-3 < E < 3$ for about 80% of the predictions). Also, overestimation and underestimation become equally likely (the CDF of E is practically symmetric). Despite the benefits of knowing $\tilde{T}$ and $\tilde{p}$, the prediction errors are still significant: more than half of the prediction errors are still larger than a factor of two. These prediction errors can be attributed to the fact that TCP samples the RTT and loss rate processes in an adaptive way, rather than periodically. In terms of our notation, the remaining prediction errors are due to the differences between $\tilde{T}$ and T, and between $\tilde{p}$ and p.
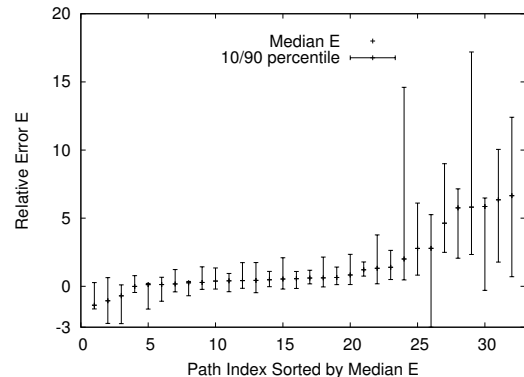

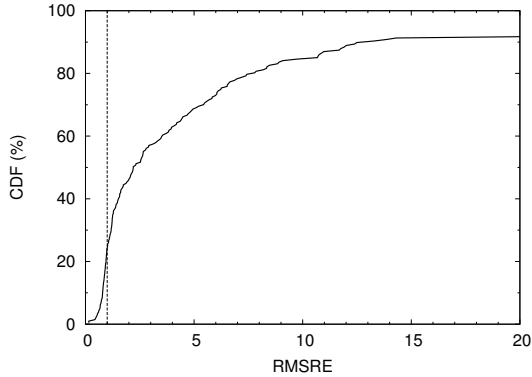Fig. 5.   Variation of the prediction error across different paths.

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[3]For W=1MB, we have $\hat{A} < W/T$ in all paths.

Fig. 6. CDF of per-trace RMSRE.

flows do not attempt to saturate the network path. To answer this question, we extended each epoch with another IPerf TCP transfer with W=20KB, and performed one experiment on each path. We verified that this transfer was window-limited on 18 of the 35 paths, and the ratio $W/(\hat{T}\hat{A})$ varied between 0.02 to 0.81. Figure 7 compares the RMSRE between the transfers with a large maximum window (W=1MB) and a small maximum window (W=20KB). Note the log-scale of the Y-axis. In all paths, the prediction error of window-limited flows was lower, often by a large factor. In particular, 14 out of the 18 paths have an RMSRE that is less than 1.0 for window-limited flows. We anticipate that for many applications a TCP throughput prediction that is accurate within a factor of two would be adequate. For such applications, an FB predictor may be appropriate as long as the transfer does not attempt to saturate the underlying available bandwidth.

C. Summary

The results of this section showed that FB prediction can be inaccurate, mostly in lossy paths and when the target flow saturates the underlying path. The major cause of prediction errors is that the RTT and/or loss rate before the transfer are significantly different than while the transfer is in progress. We note again that this cause of prediction errors is not specific to the PFTK formula. This implies that it is unlikely that other TCP throughput models would have produced more accurate FB predictions. Other important causes of prediction errors are the difference between periodic and TCP sampling of the RTT and loss rate processes, and the difference between TCP throughput and available bandwidth.

Our results also suggest that more sophisticated techniques that estimate the RTT and loss rate to be experienced by the target flow can significantly improve FB prediction. Such techniques, however, should take into account the load that will be exerted by the target flow, and the impact of that load on the queueing delays and losses in the path. More information about the underlying path, such as the capacity, available bandwidth, buffer size, number of competing flows, etc, may help achieving this goal.

Variation of prediction error across different paths and traces: Figure 5 shows the median, as well as the 10/90-th percentiles, of the relative prediction error on a per path basis (recall that we have 7×150 measurements from each path). There are three paths that we did not include in this graph because they have excessive prediction errors. With the exception of 4-5 paths that mostly give small underestimation errors, most paths give only overestimation errors. Another interesting point is that *different paths exhibit widely different predictability*. About 10 out of the 35 paths have much larger ranges of prediction error than the rest, extending up to E=10 or larger. This implies that, not only is it hard to predict TCP throughput with an FB method, but also it is hard to bound the prediction error that should be anticipated. We will return to the reasons behind this large variation of the prediction error across different paths in § VII.

Figure 6 shows the distribution of the per-trace RMSRE. Recall that we calculate a single RMSRE value for each trace (with 150 successive epochs per trace). The important observation here is that about 70% of the traces have an RMSRE that is larger than 1.0. This is a rather disappointing accuracy for a predictor, as it implies that in most cases, the prediction error will be more than a factor of two.

V. HISTORY-BASED PREDICTION

A fundamentally different approach to predicting the throughput of a large TCP transfer is to use throughput measurements of previous transfers in the same path. This *History-Based* (HB) prediction method is similar to traditional time series forecasting, where past samples of an unknown random process are used to predict the value of the process in the future. The HB approach is possible in applications where large TCP transfers are performed repeatedly over the same path.

In this section, we first introduce three families of simple linear predictors (Moving Average, Exponential Weighted Moving Average, and non-seasonal Holt-Winters). We do not examine more complex linear predictors such as ARMA or ARIMA because the selections of both their order and of their linear coefficients require a large number of past measurements [7]; instead, we expect that applications will have to perform
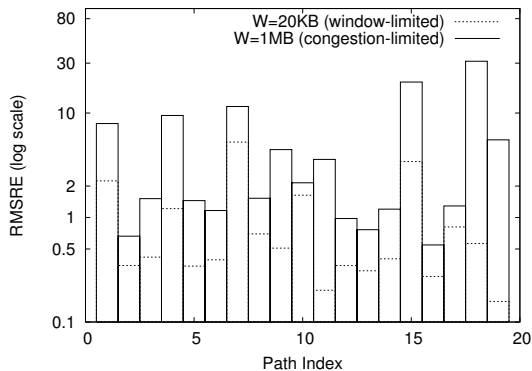

Fig. 7. Prediction accuracy for window-limited vs. congestion-limited flows.

Predictability of window-limited flows: Another interesting question is whether the FB predictor would be more accurate for window-limited flows (i.e., $W/\hat{T} < \hat{A}$), given that those

TCP throughput HB prediction based on a limited number of past transfers (say 10-20). We then show that two distinct time series "pathologies", namely *outliers* and *level shifts*, can have a major impact on the prediction error, and propose simple heuristics that can deal with these pathologies effectively.

A. Linear Predictors

- *Moving Average (MA).* Given a time series *X*, the one-step n-order MA (n-*MA*) predictor is

$$\hat{X}_{i+1} = \frac{1}{n} \sum_{k=i-n+1}^{i} X_k$$

where $\hat{X}_i$ is the predicted value and X_i is the actual (observed) value at time i. If n is too small, the predictor cannot smooth out the noise in the underlying measurements. On the other hand, if n is too large the predictor cannot aptly adapt to non-stationarities (e.g., level shifts due to load variations or routing changes).

- *Exponentially Weighted Moving Average (EWMA).* The one-step EWMA predictor is

$$\hat{X}_{i+1} = \alpha X_i + (1 - \alpha)\hat{X}_i$$

where α is the weight of the last measurement ($0 < \alpha < 1$). Similar to the MA predictor, a higher α cannot smooth out the measurement noise, while a lower α is slow in adapting to changes in the underlying time series.

- *Holt-Winters (HW).* The non-seasonal Holt-Winters predictor is a variation of EWMA that attempts to capture the *trend* in the underlying time series, if such a trend exists[4]. This predictor is more appropriate than EWMA for non-stationary processes, especially if the latter exhibit a linear trend. A non-seasonal HW predictor maintains a separate smoothing component $\hat{X}_i^s$ and a trend component $\hat{X}_i^t$, and it depends on two parameters α and β, both in $(0, 1)$. Specifically, the predicted value at time i is

$$\hat{X}_i^f = \hat{X}_i^s + \hat{X}_i^t,$$

where

$$\hat{X}_{i+1}^s = \alpha X_i + (1 - \alpha)\hat{X}_i^f,$$

$$\hat{X}_{i+1}^t = \beta(\hat{X}_i^s - \hat{X}_{i-1}^s) + (1 - \beta)\hat{X}_{i-1}^t.$$

The initial values of $\hat{X}^s$ and $\hat{X}^t$ are X_0 and $X_1 - X_0$, respectively, assuming that the time series starts at $i=0$.

B. Detection of Level Shifts and Outliers

While experimenting with various predictors, we found out that the largest prediction errors are often caused by level shifts and outliers in the observed time series. Furthermore, if we manage to somehow avoid these two characteristics in the throughput time series, then the exact choice of the predictor, or of its parameters, does not make a significant difference.

A level shift is a type of non-stationarity, and it causes a significant and typically sudden change in the mean of the

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[4]The forecasting literature sometimes refers to this non-seasonal version of the HW predictor as Holt's predictor.

observed time series. An outlier is a measurement that is significantly different, beyond the typical level of statistical variations, relative to nearby measurements. Both outliers and level shifts have been studied extensively in the theory of forecasting [28]. In Figures 8(a), 8(b) and 8(c) we show examples of traces that exhibit both outliers and level shifts, observed in our TCP throughput measurements. One way to deal with level shifts, after they are detected, is to restart the predictor, ignoring all previous history. Outliers, on the other hand, can be just ignored.

We next describe simple heuristics to detect level shifts and outliers. Suppose that $\{X_1, ...X_n\}$ is the sequence of past measurements, ignoring outliers, where X_1 is the first measurement after the last detected level shift. We determine that the measurement X_k is an increasing (decreasing) level shift if it satisfies the following three conditions:

1) The measurements $\{X_1, ...X_{k-1}\}$ are all lower (higher) than the measurements $\{X_k, ...X_n\}$,
2) The median of $\{X_1, ...X_{k-1}\}$ is lower (higher) than the median of $\{X_k, ...X_n\}$ by more than a relative difference χ, and
3) $k + 2 \leq n$.

The last condition aims to avoid misinterpreting an outlier as a level shift. Upon the detection of a level shift, we ignore all measurements prior to X_k and restart the predictor from X_k. On the other hand, a measurement X_k (with $k < n$) is considered an outlier if it differs from the median of the measurements in $\{X_1, ...X_n\}$ by more than a relative difference of ψ. Outliers are discarded from the history of previous measurements.

Figures 8(d), 8(e) and 8(f) show the RMSRE for the three sample traces with five different predictors: MA, MA-LSO, EWMA, HW, and HW-LSO. The *LSO* acronym is used when we use the previous heuristics for the detection of Level Shifts and Outliers. For the MA and MA-LSO predictors, we show results for four different values of n (see the figure's legend). For the EWMA and HW predictors, we show results for three values of α. We observed that, at least for our datasets, the RMSRE does not strongly depend on β, χ and ψ. We found empirically that the following values perform reasonably well, in terms of minimizing the RMSRE, at least in our datasets: β=0.2, χ=0.3, and ψ=0.4. On the other hand, the parameters n and α play a major role in the prediction accuracy when the LSO heuristic is *not* used. The LSO heuristic decreases the prediction error significantly, and makes the predictors more robust to the choice of n or α. The difference between the accuracy of MA-LSO and HW-LSO is not major, although the latter tends to perform slightly better. More results for the accuracy of these two predictors will be given in the next section.

VI. HB PREDICTION ACCURACY

In this section, we apply the HB predictors of the previous section to the measurements described in § IV. Our objective is to compare the most promising HB predictors that we experimented with, and to examine how the HB prediction accuracy varies in different paths, with window-limited flows, and with different transfer frequencies.

(a) Level Shift

(b) Trend, Level Shift, & Outliers

(c) Level Shift & Outliers

(d) Prediction error for Trace (a)

(e) Prediction error for Trace (b)
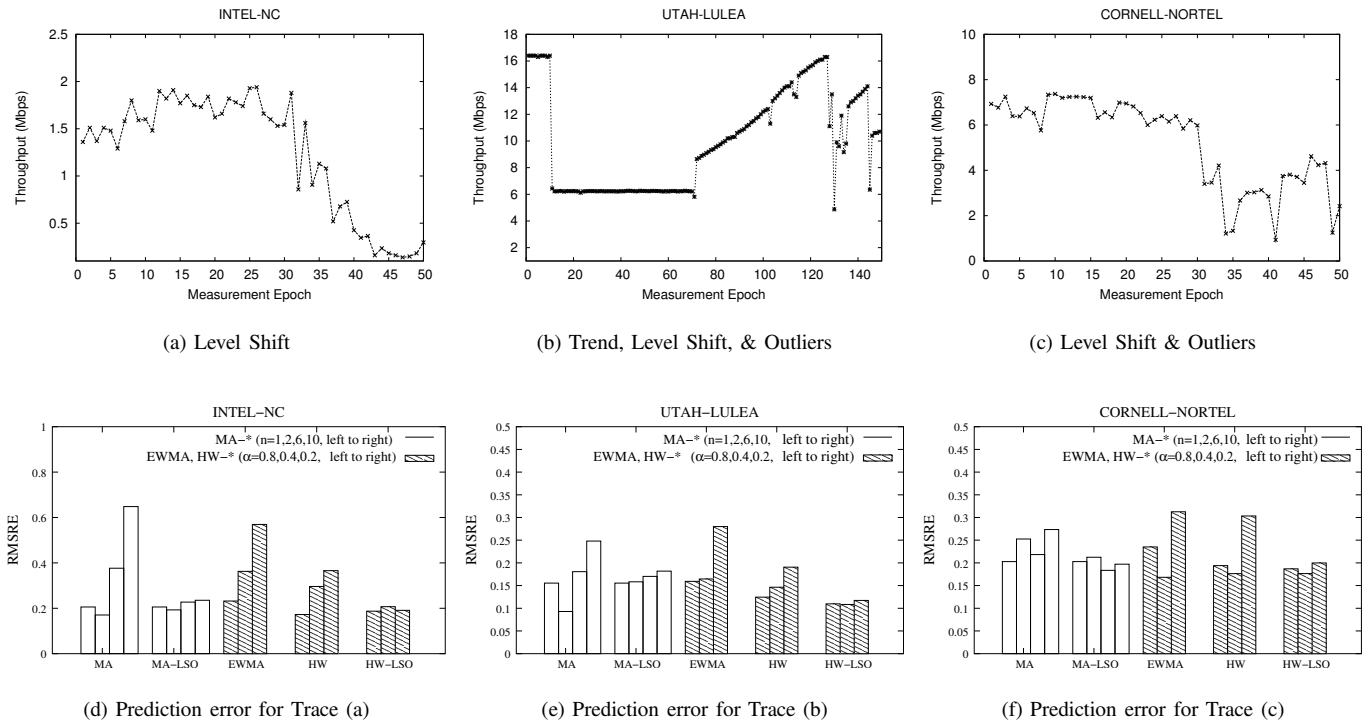
(f) Prediction error for Trace (c)

Fig. 8.   Examples of TCP throughput traces and the prediction errors (RMSRE) with various predictors.

A. Results

Comparison of HB predictors: Figures 9 and 10 summarize the prediction error (in terms of RMSRE) of several MA and HW predictors, respectively. The EWMA predictor performs similarly to HW. Without LSO, the n-MA predictors perform very similarly when $n < 20$ (we do not show all of them), except the trivial case of $n=1$ that performs slightly worse. With LSO, there is a significant reduction in the RMSRE of MA predictors. For HW predictors, $\alpha=0.8$ (0.8-HW) performs visibly better than $\alpha=0.4$. Further experimentation showed that $\alpha=0.8$ is close to the optimal for our dataset, and we use this value for HW predictor hereafter unless otherwise noted. HW predictor is also significantly improved with LSO. A comparison of MA-LSO (with $n=10$) and HW-LSO shows that the accuracy of the latter is only slightly better. This is an indication that not many of our traces exhibit linear trends.

Comparison of FB and HB predictors: Even though these two classes of predictors are complementary, in some cases it may be possible to use either FB or HB predictor. Comparing the RMSRE of the FB predictor (see Figure 6) with that of the HB predictors, we can see that the accuracy of the latter is dramatically better. Specifically, HB predictors give RMSRE less than 0.4 for about 90% of the traces. The same RMSRE percentile for the FB predictor is 20, while the median RMSRE is about 2. One may argue that this comparison is not fair for FB, since FB is applicable without any knowledge of previous TCP transfer throughput measurements. If it is possible to collect and use such historical data, however, this comparison shows that HB prediction should be preferred to FB prediction.

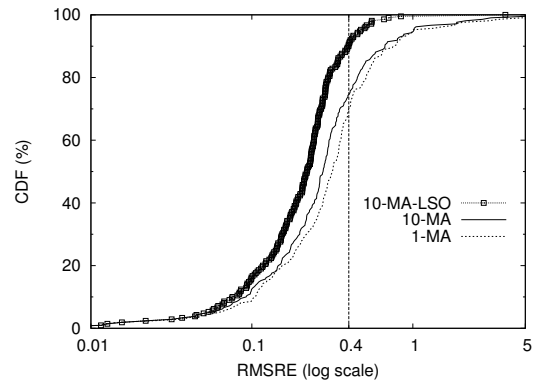**RMSRE vs. CoV of throughput measurements:** We are

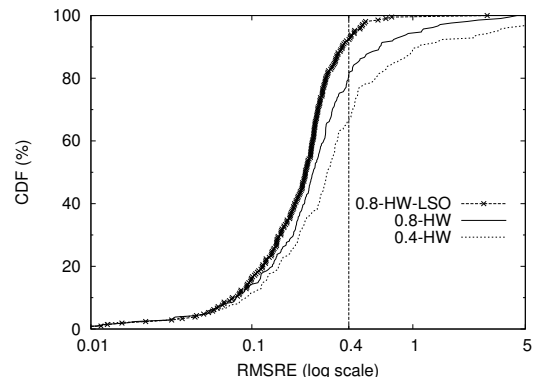Fig. 9.   Exponential Weighted Moving Average prediction error.

Fig. 10.   Holt-Winters prediction error.

interested in the relation between the prediction RMSRE for a given trace and the Coefficient of Variation (CoV) of the corresponding TCP throughput time series[5]. The reason for this comparison will become clear in the following section, where we use the CoV as an indication of the TCP throughput *predictability* in a path. To calculate the CoV of a trace, we isolate stationary periods based on the detected level shifts and exclude outliers. We then calculate the weighted average of the CoVs for different periods (with the weight of each period being the number of corresponding measurements). In the RMSRE calculations, we also exclude measurements that were identified as outliers. Figure 11 shows the correlation of the CoV and RMSRE for each collected trace, using the HW-LSO predictor. Note the strong correlation between the two metrics. The correlation coefficient between them is 0.91. We can thus assume, at least as a first-order approximation, that the RMSRE prediction error with HW-LSO is equal to the CoV of the corresponding time series, at least in the datasets we experimented with.
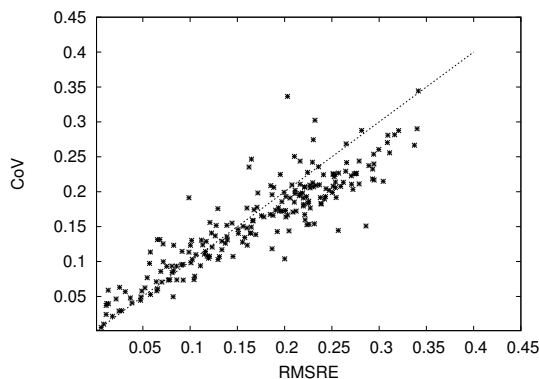

Fig. 11.   Prediction error versus CoV.

Variations in path predictability: Figure 12 provides close-up views of the accuracy of several predictors in 12 sample paths. We classify these paths into four representative classes (described in the figure's caption), based on the average prediction error as well as the variation of the error across different traces in the same path. Each subfigure represents a specific path, with the X-axis numbers indicating different traces. For each trace, successive bars show the RMSRE with 1-MA, 10-MA, HW, and HW-LSO, from left to right. As previously noted, *the HW-LSO predictor is almost always the best in terms of RMSRE*. A more important observation from these graphs, however, is that *there are major differences in the prediction error between different paths*. Some paths have quite low RMSRE and they are fairly predictable, others have larger RMSRE but the RMSRE is quite stable (predictable errors), while others have either large RMSRE variations (un-predictable errors), or high RMSRE (unpredictable through-put). What causes different paths to behave so differently in terms of their TCP throughput predictability? We focus on this important question in the next section.

Predictability of window-limited flows: When the target

[5]Recall that the CoV is the ratio of the standard deviation to the mean.

flow is window-limited, it would probably not be subject to the dynamic variations of the available bandwidth, and so we can expect higher predictability. Figure 13 compares the prediction error for window-limited flows (W=20KB) and for congestion-limited flows (W=1MB), using the same traces as in Figure 7. Notice that *window-limited flows have a lower RMSRE, confirming the insight that the throughput is more predictable when the target flow does not attempt to saturate the path*. The RMSRE difference is not always major, however, especially when the RMSRE for congestion-limited flows is already low (around 0.1). These remaining errors are probably due to short-term load variations in the underlying path, or random packet losses that the target flow experiences, causing unavoidable variations in the resulting TCP throughput, independent of W.
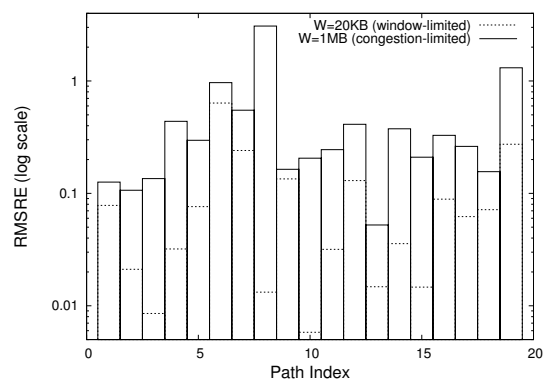

Fig. 13.   Prediction error for window-limited vs. congestion-limited flows.

The effect of the target flow frequency: All previous results are based on periodic TCP transfers, performed approximately every 3 minutes. We expect the prediction accuracy to depend on this "TCP transfer period". A time series with a larger period spans a wider history horizon, and so route changes or major load variations become more likely.
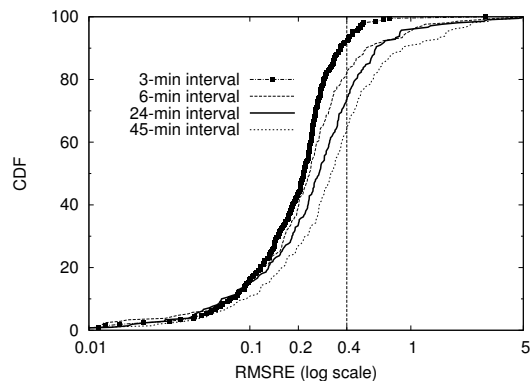

Fig. 14.   Prediction error with different TCP transfer periods.

To see how the measurement period affects the prediction error, we down-sample the original traces at different frequencies. We then apply the HW-LSO predictor to the down-sampled traces, producing RMSRE of predictions for transfer periods of 6, 24, and 45 minutes. Figure 14 shows the
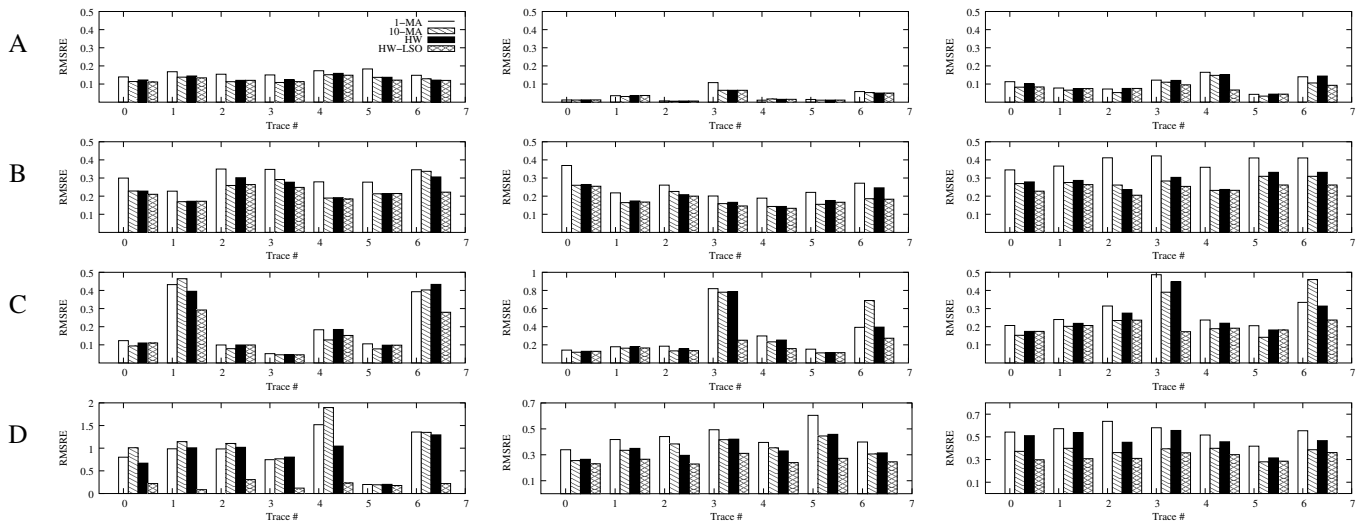
Fig. 12. A: Predictable paths (low RMSRE), B: Paths with small and predictable errors (stable RMSRE), C: Paths with small but unpredictable errors (varying RMSRE), D: Unpredictable paths (high RMSRE, notice the different Y-axis ranges).

results. As we would expect, *the prediction accuracy degrades as we increase the measurement period*. Fortunately, though, the prediction errors remain reasonable even with the largest measurement period. Specifically, with the 45-min period, 65% of the traces have an RMSRE below 0.4. At the 90-th percentile of the traces the RMSRE is less than 0.4 with the 3-min period, and less than 1.0 with the 45-min period. This is an encouraging result, as it implies that *HB prediction is fairly accurate even if it relies only on sporadic previous TCP transfers, every few minutes, on the given paths*. Of course we emphasize once more that this conclusion is based only on our dataset, and it is possible that other Internet paths behave differently.

B. Summary

This section has evaluated the accuracy of HB prediction with respect to several factors, some of which have not been examined before. Specifically, we have shown that:

1) Even a limited history of sporadic previous TCP transfers is often sufficient to achieve a fairly good prediction accuracy.
2) Simple heuristics to detect outliers and level shifts can significantly reduce the number of large prediction errors.
3) If HB prediction is feasible, i.e., if there is a history of previous TCP transfers in the same path, then HB prediction is more accurate than FB prediction.
4) Different paths can exhibit distinct patterns of prediction accuracy. Consequently, even with the same prediction algorithm and available history, the resulting accuracy can be significantly different from path to path.
5) The predictability of an HB predictor is higher when the transfer is window-limited. Consequently, if predictability is more important than throughput maximization, then the TCP flow should have a limited advertised window such that it does not saturate the underlying path.

VII. TWO PREDICTABILITY FACTORS

The empirical results in the previous section raise the following question: *what makes TCP throughput much less predictable in some network paths than in others?* In this section, we focus on this question and identify two major factors that affect the accuracy of HB prediction in a path: load and the degree of multiplexing. Specifically, we rely on simple queueing models that provide a framework for reasoning about the relationship between TCP throughput predictability and the previous two factors.

First, we focus on the connection between the relative prediction error and the Coefficient of Variation (CoV) of a given time series. Consider a second-order stationary time series X with mean μ_X, variance σ_X^2, and covariance $\gamma_X(k)$. According to the Yule-Walker forecasting model [23], an autoregressive one-step predictor based on the n most recent samples of X has the following prediction error variance:

$$\text{Var}[e_n] = \text{Var}[X_{n+1} - \hat{X}_{n+1}] = \sigma_X^2 - \sum_{k=1}^{n} a_{X,n}(k)\gamma_X(k)$$

where X_i and $\hat{X}_i$ are the actual and predicted values of X, respectively, at time i, and $\{a_{X,n}(i), i = 1, \ldots n\}$ are the autoregressive coefficients of X that minimize the mean square prediction error. The corresponding relative prediction error, in terms of the Normalized Root Mean Square Error (NRMSE) [6] is given by:

$$\frac{\sqrt{\text{Var}[e_n]}}{\mu_X} = \frac{\sqrt{E[e_n^2]}}{\mu_X} = \sqrt{\text{CoV}_X^2 - \frac{\sum_{k=1}^{n} a_{X,n}(k)\gamma_X(k)}{\mu_X^2}}, \tag{7}$$

where $\text{CoV}_X = \sigma_X/\mu_X$. The key point here is that *the relative prediction error increases with the CoV of the underlying time series*. Also, if the time series has a weak correlation structure

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[6]Notice that although NRMSE is not exactly the same as RMSRE, they are reasonably close as long as μ_x does not vary significantly, say spanning an order of magnitude, in a time series. This is true for most of the paths we measured.

then the relative prediction error is approximately equal to the time series CoV,

$$\text{if } \gamma_X(k) \approx 0, \text{ then NRMSE} = \frac{\sqrt{E[e_n^2]}}{\mu_X} \approx \text{CoV}_X \quad (8)$$

Also recall the observation from Figure 11: the RMSRE with the HW-LSO predictor and the CoV of the corresponding time series are approximately equal. Consequently, in the following we are interested in the effects of load and degree of multiplexing on the CoV of the TCP throughput time series, rather than examining directly the effect of these factors on the RMSRE or on the NRMSE.

A. Effect of Load

Consider a link of capacity C, modeling the bottleneck of a path. We next examine the effect of that link's load conditions through two different models: first, an Independent and Identically Distributed (IID) process for the aggregate traffic at a bufferless server, and second, a Poisson process of IID session arrivals at a Processor Sharing server.
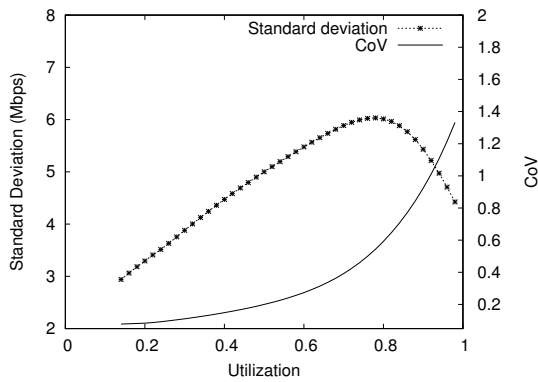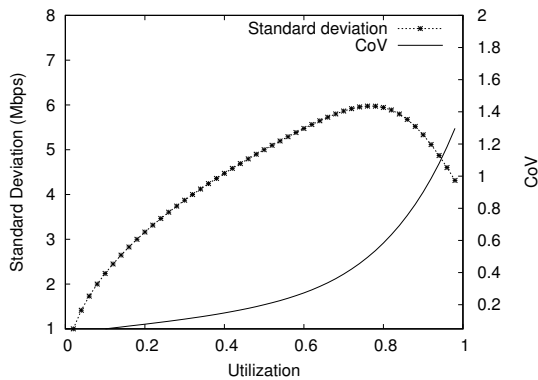

Fig. 15. Gaussian process.

Fig. 16. Poisson process.

1) IID arrival process at bufferless server: Suppose that the *arriving* traffic rate at a given time scale T can be modeled as an IID process Y. Without loss of generality, $T=1$ time unit. Let Z be the *observed* traffic rate at the output of the link at
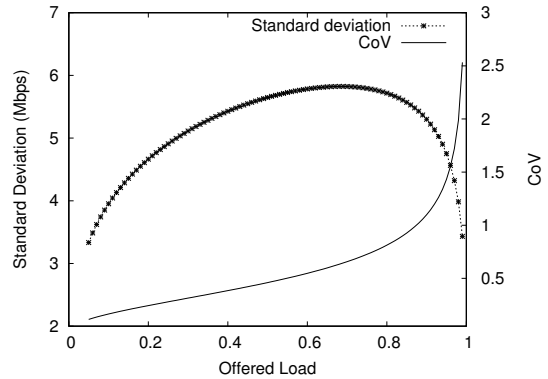

Fig. 17. Processor Sharing model.

the same time scale. For a bufferless link, the observed rate process is given by

$$Z = \begin{cases} Y & \text{if } Y < C \\ C & \text{if } Y \geq C \end{cases} \quad (9)$$

and so the probability distribution function of Z can be obtained from that of Y. The available bandwidth is given by $A = C - Z$, and its CoV is

$$\text{CoV}(A) = \frac{\sqrt{\text{Var}[Z]}}{C - E[Z]}$$

If we assume that the TCP throughput is, as a first-order approximation, equal to the available bandwidth, then the previous expression also gives the TCP throughput CoV.

We used Mathematica to derive $\text{CoV}(A)$ for two offered load processes Y: a Gaussian process and a Poisson process. The resulting $\text{CoV}(A)$, as well as the std-deviation of A, are shown in Figures 15 and 16, respectively, as a function of the link utilization $\rho = (C - A)/C$. The key observation is that *the CoV of the available bandwidth increases with the link utilization*. If the TCP throughput follows the variations of the available bandwidth, then based on (8) we should expect *a higher relative prediction error under heavier load conditions*.

As an interesting side-note, note that the standard deviation reaches a maximum as ρ increases, and then it drops. The reason for that drop is that, in heavy-load conditions, the link is almost always utilized and so there is little *absolute* variation in the available bandwidth. This point has been studied in more depth by Tian et al. in [33]. In relative terms, however, the variability of A increases monotonically with ρ, as shown by the CoV curve.

2) Processor Sharing model with Poisson session arrivals: The previous model does not capture what happens at a congested link, in which the available bandwidth is zero. In this paragraph, we model the traffic as a stream of IID sessions arriving at a link, based on a Poisson process with average rate λ. The mean size of the sessions is θ. The normalized offered load is $\rho = (\lambda\theta)/C$. Furthermore, we model the link as a Processor Sharing server, meaning that if there are N sessions in the link then their instantaneous service rate is $r(N)=C/N$. Since the available bandwidth is zero when the link is not idle, this a more appropriate model for a congested link [13]. An

arriving session, modeling the target flow, will obtain the same throughput $r(N)$ as any other active flow. So, in this model, we are not interested in the CoV of the available bandwidth, but in the CoV of the per-flow throughput $r(N)$.

The probability distribution for the number of active flows N in the above Processor Sharing model is given by

$$\pi(N) = \rho^N(1 - \rho)$$

We again use Mathematica to derive the CoV of the target flow's throughput $r(N)$:

$$\mathrm{CoV}[r(N)] = \frac{(1-\rho)\log(1-\rho)^2 + \rho \cdot L(2,\rho)}{(\rho-1)\log(1-\rho)^2}$$

where $L(n,x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}$. Figure 17 shows the standard deviation and CoV of $r(N)$ as a function of the offered load ρ. The main observation is the same as in the IID traffic model: *the CoV of a flow's throughput increases with the offered load ρ, implying that we should expect a higher relative prediction error under heavier load conditions.*

B. Effect of Degree of Multiplexing

The conventional wisdom is that network traffic is "smoother" in links with a higher degree of multiplexing, i.e., with a larger number of simultaneously active flows. Using a simple queuing model, we aim to better understand this insight, and the conditions under which it is valid.

Consider again a model of Poisson session arrivals. Instead of the Processor Sharing model (which leaves no available bandwidth), suppose that sessions are rate limited, and for simplicity, the rate for each session is constant and equal to r. The number of sessions N on the link follows a Poisson distribution with mean and variance $E[N] = \mathrm{Var}[N] = (\lambda\theta)/r$ [13].

The utilized link capacity at any point in time is $Y = Nr$, with mean $E[Y] = rE[N] = \lambda\theta = \rho C$, and variance $\mathrm{Var}[Y] = r^2\mathrm{Var}[N]$. So, the CoV of the available bandwidth is

$$\mathrm{CoV}[A] = \mathrm{CoV}[C - Y] = \frac{1}{\sqrt{E[N]}}\frac{\rho C}{C(1 - \rho)} \qquad (10)$$

Suppose that we keep the utilization ρ constant, but decrease the session service rate r so that the average number of sessions $E[N]$ increases. Equation (10) shows that the CoV of A decreases with the square root of $E[N]$. This confirms that *we should expect a lower relative prediction error as the number of competing flows on the link increases, but only when the utilization remains constant.*

C. Summary

This section used some simple queueing models to confirm the following insights:

- the relative prediction error increases with the CoV of the underlying time series,
- the CoV of the available bandwidth process in a non-congested link, or the CoV of a flow's throughput in a congested link, increases with the offered load on that link,

- the CoV of the available bandwidth process decreases with the number of competing flows on the link, if the utilization remains constant.

Obviously, our models are based on quite restrictive assumptions and they do not consider the many idiosyncrasies of TCP. In particular, the previous analysis assumed that the TCP throughput follows the variability of the available bandwidth at the bottleneck link of its path. This assumption is obviously not true in short time scales (less than a few RTTs), and so the previous insights may not be true for short TCP flows. We note that we have also validated the previous conclusions in the case of long TCP transfers with simulations of both TCP and non-TCP cross traffic.

VIII. CONCLUSIONS

This paper investigated two classes of throughput predictors for large TCP transfers. FB prediction is an attractive option, given that it does not require intrusive measurements or any history of prior TCP transfers. We demonstrated however that it can be inaccurate, especially when the transfer attempts to saturate the path, and we explained the reasons for these errors. HB prediction, on the other hand, is quite accurate but it is feasible only when there is a history of previous TCP transfers in the same path. Although the accuracy of HB prediction does not depend so much on the actual predictor, it does depend on the transfer's maximum congestion window size and on the underlying path. We explained the path dependency based on two factors: the load and the degree of multiplexing on the bottleneck link of the path.

In future work, it would be interesting to examine hybrid predictors, which rely on TCP models as well as on recent history. Another direction would be to develop TCP throughput models that are specifically designed for prediction, and that take as inputs various estimates of the path load, buffering, and cross traffic nature. In terms of HB prediction, more complex predictors (such as ARIMA models) can be also evaluated, even though our measurements indicate that the prediction error is already quite low, probably for any practical purposes, in most paths.

REFERENCES

[1] Resilient Overlay Network (RON). http://nms.lcs.mit.edu/ron/, February 2005.
[2] A. Akella, B. Maggs, S. Seshan, A. Shaikh, and R. Sitaraman. A Measurement-Based Analysis of Multihoming. In *Proceedings of ACM SIGCOMM*, 2003.
[3] A. Akella, J. Pang, A. Shaikh, B. Maggs, and S. Seshan. A Comparison of Overlay Routing and Multihoming Route Control. In *Proceedings of ACM SIGCOMM*, 2004.
[4] D. Andersen, H. Balakrishnan, F. Kaashoek, and R. Morris. Resilient Overlay Networks. In *Proceedings of ACM Symposium on Operating Systems Principles*, October 2001.
[5] D. G. Andersen, A. C. Snoeren, and H. Balakrishnan. Best-Path vs. Multi-Path Overlay Routing. In *Proceedings Internet Measurement Conference (IMC)*, pages 91–100, 2003.
[6] F. Baccelli and K. B. Kim. TCP Throughput Analysis under Transmission Error and Congestion Losses. In *Proceedings of IEEE INFOCOM*, 2004.
[7] G. E. P. Box, G. M. Jenkins, and G. C. Reinsel. *Time Series Analysis: Forecasting and Control (3rd edition)*. Prentice-Hall, 1994.
[8] N. Cardwell, S.Savage, and T.Anderson. Modeling TCP Latency. In *Proceedings of IEEE INFOCOM*, March 2000.

[9] Bram Choen. Incentives Build Robustness in BitTorrent. http://bitconjurer.org/BitTorrent/bittorrentecon.pdf, May 2003.

[10] Y-H. Chu, S. G. Rao, S. Seshan, and H. Zhang. Enabling Conferencing Applications on the Internet using an Overlay Multicast Architecture. In *Proceedings of ACM SIGCOMM*, August 2001.

[11] S. Floyd, M. Handley, J. Padhye, and J. Widmer. Equation-Based Congestion Control for Unicast Applications. In *Proceedings of ACM SIGCOMM*, 2000.

[12] I. Foster. *The Grid: Bluepring for a New Computing Infrastructure*. Morgan Kaufmann, 2004 (2nd edition).

[13] S. Ben Fredj, T. Bonald, A. Proutiere, G. Regnie, and J. W. Roberts. Statistical Bandwidth Sharing: A Study of Congestion at Flow Level. In *Proceedings of ACM SIGCOMM*, August 2001.

[14] M. Garetoo, R. L. Cigno, M. Meo, and M. Marson. A Detailed and Accurate Closed Queueing Network Model of Many Interacting Flows. In *Proceedings of IEEE INFOCOM*, 2001.

[15] M. Goyal, R. Guerin, and R. Rajan. Predicting TCP Throughput From Non-invasive Network Sampling. In *Proceedings of IEEE INFOCOM*, 2002.

[16] N. Hu and P. Steenkiste. Evaluation and Characterization of Available Bandwidth Probing Techniques. *IEEE Journal on Selected Areas in Communications*, 21(6):879–894, August 2003.

[17] Iperf. http://dast.nlanr.net/Projects/Iperf/.

[18] M. Jain and C. Dovrolis. End-to-End Available Bandwidth: Measurement Methodology, Dynamics, and Relation with TCP Throughput. *IEEE/ACM Transactions on Networking*, 11(4):537–549, August 2003.

[19] B. Krishnamurthy, C. Wills, and Y. Zhang. On the Use and Performance of Content Distribution Networks. In *Proceedings of ACM SIGCOMM Internet Measurement Workshop*, OCT 2001.

[20] M. Mathis, J. Semke, and J. Madhavi. The Macroscopic Behavior of the TCP Congestion Avoidance Algorithm. *ACM Computer Communications Review*, 27(3):67–82, July 1997.

[21] T. S. Eugene Ng, Y h. Chu, S. G. Rao, K. Sripanidkulchai, and H. Zhang. Measurement-Based Optimization Techniques for Bandwidth-Demanding Peer-to-Peer Systems. In *Proceedings of IEEE INFOCOM*, 2003.

[22] J. Padhye, V.Firoiu, D.Towsley, and J. Kurose. Modeling TCP Throughput: A Simple Model and its Empirical Validation. *IEEE/ACM Transactions on Networking*, 8(2):133–145, April 2000.

[23] M. Pourahmadi. *Foundations of Time Series Analysis and Prediction Theory*. John Wiley and Sons, 2001.

[24] Y. Qiao, J. Skicewicz, and P. Dinda. An Empirical Study of the Multiscale Predictability of Network Traffic. In *IEEE Proceedings of HPDC*, 2003.

[25] L. Qiu, V. N. Padmanabhan, and G. M. Voelker. On the Placement of Web Server Replicas. In *Proceedings of IEEE INFOCOM*, pages 1587–1596, 2001.

[26] S. Ratnasamy, M. Handley, R. Karp, and S. Shenker. Topologically-Aware Overlay Construction and Server Selection. In *Proceedings of IEEE INFOCOM*, 2002.

[27] V. Ribeiro, R. Riedi, R. Baraniuk, J. Navratil, and L. Cottrell. pathChirp: Efficient Available Bandwidth Estimation for Network Paths. In *Proceedings of Passive and Active Measurements (PAM) workshop*, April 2003.

[28] R.S.Tsay. Outliers, Level Shifts, and Variance Changes in Time Series. *Journal of Forecasting*, 1988.

[29] A. Sang and S. Li. A Predictability Analysis of Network Traffic. *Computer Networks*, 39(4):329 – 345, Jul 2002.

[30] B. Sikdar, S. Kalyanaraman, and K. S. Vastola. Analytic Models for the Latency and Steady-State Throughput of TCP Tahoe, Reno and SACK. *IEEE/ACM Transactions on Networking*, 11(6):959–971, December 2003.

[31] J. Strauss, D. Katabi, and F. Kaashoek. A Measurement Study of Available Bandwidth Estimation Tools. In *Proceedings of ACM Internet Measurement Conference*, 2003.

[32] M. Swany and R. Wolski. Multivariate Resource Performance Forecasting in the Network Weather Service. In *Proceedings of Supercomputing*, 2002.

[33] X. Tian, J. Wu, and C. Ji. A Unified Framework for Understanding Network Traffic Using Independent Wavelet Models. In *INFOCOM*, 2002.

[34] S. Vazhkudai, J. Schopf, and I. Foster. Predicting the Performance of Wide Area Data Transfers. In *Proceedings of IEEE IPDPS*, 2002.

[35] S. Vazhkudai and J. Schopt. Predicting Sporadic Grid Data Transfers. In *Proceedings of IEEE HPDC*, 2002.

[36] R. Wolski, N. Spring, and J. Hayes. The Network Weather Service: A Distributed Resource Performance Forecasting Service for Metacomputing. *Journal of Future Generation Computing Systems*, 1998.

[37] R. Wolski, N. Spring, and C. Peterson. Implementing a Performance Forecasting System for Metacomputing: the Network Weather Service. In *Proceedings of Supercomputing*, 1997.

[38] E. Zegura, M. Ammar, Z. Fei, and S. Bhattacharjee. Application-Layer Anycasting: A Server Selection Architecture and Use in a Replicated Web Service. *IEEE/ACM Transactions on Networking*, 8:455, 2000.

[39] Y. Zhang, N. Duffield, V. Paxson, and S. Shenker. On the Constancy of Internet Path Properties. In *Proceedings of ACM SIGCOMM Internet Measurement Workshop*, pages 197–211, November 2001.

APPENDIX: MAIN SYMBOLS

T	RTT experienced by target flow
$\hat{T}$	RTT measured with periodic probing prior to target flow
$\tilde{T}$	RTT measured with periodic probing during target flow
p	loss rate experienced by target flow
$\hat{p}$	loss rate measured with periodic probing prior to target flow
$\tilde{p}$	loss rate measured with periodic probing during target flow
p'	congestion event probability experienced by target flow
R	actual throughput of target flow
$\hat{R}$	predicted throughput of target flow
$\tilde{R}$	expected throughput of target flow based on $\tilde{T}$ and $\tilde{p}$
$\hat{A}$	available bandwidth measured prior to target flow
W	maximum window of target flow